A GENERALIZED DIFFUSION THEORY OF BINARY HOMOGENEOUS MIXTURES.

The motion of single-temperature binary homogeneous mixtures is studied by using a generalized diffusion theory.

A generalized diffusion theory was proposed in [1] to describe the motion of homogeneous n-component mixtures of different temperatures. The fundamental equations were obtained to determine some characteristic velocity of a mixture $u_{a}$ of the diffusion fluxes referred to this velocity, and other parameters.

The equations obtained are derived and studied in this paper in the case of singletemperature binary homogeneous mixtures for different characteristic velocities $u_{\alpha}$. The motion of a liquid binary mixture within an infinite rotating cylinder is examined as a simple example.

## 1. Fundamental Equations of Motion of Binary Homogeneous Mixtures

Let there be a mixture consisting of two components with the densities $\rho_{1}$ and $\rho_{2}$, with the number of moles $N_{1}$ and $N_{2}$, with the molar masses $M_{1}$ and $M_{2}$ and with the molar partial volumes $V_{1}$ and $V_{2}$. Let us assume that a common temperature $T$ is set in the mixture and there is no chemical reaction between the components.

Let us study the general motion of the mixture by using a certain characteristic velocity $u_{a}$, and the relative motions of the components by using generalized diffusion fluxes $\mathrm{J}_{\mathrm{k}}^{a}$. In practice, it is known that the "choice of a convenient reference system which governs what is to be understood by the common mixture velocity can greatly facilitate the computation and interpretation of the results" [2]. One of the following characteristic velocities is of ten used for homogeneous mixtures: the mean mass, volume, molar flow rate, and the velocity of one of the components. Depending on the specific conditions, some behavior of one of these velocities, e.g., its magnitude $[2,3]$ or direction, can be "predicted" in many practical problems. This explains why the mean mass flow rate of a mixture is not always most convenient for the solution of problems. It is hence necessary to use other characteristic velocities to formulate the fundamental laws of mixture motion.

The main system of equations to determine $u_{a}, J_{k}^{\alpha}$, and $T$ has the form [1]

$$
\left.\left.\begin{array}{c}
u_{a}=\sum_{k=1}^{2} a_{k} u_{k}, \quad J_{1}^{a}=\rho_{1}\left(u_{1}-u_{a}\right), \quad J_{2}^{a}=-\left(\rho_{2} a_{1} / a_{2} \rho_{1}\right) J_{1}^{a}, \\
\frac{\partial \rho_{k}}{\partial t}+\bar{\nabla} \cdot\left(\rho_{k} u_{a}\right)=-\nabla \cdot J_{k}^{a}, \quad \frac{\partial \rho}{\partial t}+\nabla \cdot\left(\rho u_{a}\right)=-\nabla \cdot \sum_{k=1}^{2} J_{k}^{a}, \\
\rho \frac{d^{(a)} u_{a}}{d t}=\rho f-\nabla p+\nabla \cdot \tau_{a}-\sum_{k=1}^{2} \frac{D^{(a)} J_{k}^{a}}{D t}, \quad \tau_{a}=\frac{\lambda_{a}}{T}\left(\nabla \cdot u_{a}\right) I+\frac{2 \mu_{a}}{T} e_{a},  \tag{1.1}\\
-\frac{D^{(a)} J_{1}^{a}}{D t}=Q_{1}^{a}=-T \frac{\rho_{1}^{2} a_{2}^{2}}{a_{11}^{a}\left(\rho_{1} a_{2}^{2}+\rho_{2} a_{1}^{2}\right)}\left\{J_{1}^{a}+\left[\alpha_{1}^{a}-\alpha_{11}^{a}\left(h_{1}-\right.\right.\right. \\
a_{2} \rho_{1}
\end{array}\right)\right] \frac{\nabla T}{T^{2}}-\left(v_{1}-\frac{a_{1} \rho_{2}}{a_{2} \rho_{1}}\right) \nabla p-\frac{\alpha_{11}^{a}}{T}\left[f_{1}-\frac{a_{1} \rho_{2}}{a_{2} \rho_{1}} f_{2}-\left(1-\frac{a_{1} \rho_{2}}{a_{2} \rho_{1}}\right) \times \$\right.
$$

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$$
\begin{gathered}
\left.\left.\times \frac{d^{(a)} u_{a}}{d t}-\left(1+\frac{a_{1}}{a_{2}}\right)\left(\nabla \mu_{1}\right)+\frac{a_{1} \rho_{2}}{a_{2} \rho_{1}} \sum_{k=1}^{2} J_{k}^{a} \frac{d^{(a)}}{d t}\left(\frac{a_{k}}{\rho_{k}}\right)\right]\right\}, \\
-\left[q+\left(1-\frac{a_{1} \rho_{2}}{a_{2} \rho_{1}}\right) J_{1}^{a}\right] \doteq\left[\alpha^{a}-\alpha_{1}^{a}\left(h_{1}-\frac{a_{1} \rho_{2}}{a_{2} \rho_{1}} h_{2}\right)\right] \frac{\nabla T}{T^{2}}+ \\
+\frac{a_{1}^{a}}{T}\left\{f_{1}-\frac{a_{1} \rho_{2}}{a_{2} \rho_{1}} f_{2}-\left(1-\frac{a_{1} \rho_{2}}{a_{2} \rho_{1}}\right) \frac{d^{(a)} u_{a}}{d t}-\left(1+\frac{a_{1}}{a_{2}}\right)\left(\nabla \mu_{1}\right)_{p, T}-\right. \\
\left.-\left(v_{1}-\frac{a_{1} \rho_{2}}{a_{2} \rho_{1}} v_{2}\right) \nabla p-\left(1+\frac{\rho_{2} a_{1}^{2}}{\rho_{1} a_{2}^{2}}\right) \frac{Q_{1}^{a}}{\rho_{1}}-\frac{a_{1} \rho_{2}}{a_{2} \rho_{1}} \sum_{k=1}^{2} J_{k}^{a} \frac{d^{(a)}}{d t}\left(\frac{a_{k}}{\rho_{k}}\right)\right\} .
\end{gathered}
$$

The quantities $a_{1}, \alpha_{2}$ in (1.1) are normalized weights which we use to obtain the mean mixture velocity $u_{a} ; f_{1}, f_{2}$, mass forces acting on unit mass of each component; $h_{1}$ and $h_{2}$, enthalpies per unit mass of each component; $v_{1}$ and $v_{2}$, partial specific volumes; $p$, equilibrium pressure; $\tau a$, viscous stress tensor; $e_{a}$, strain rate tensor; $h^{*}$, heat source; and $\mu_{1}$ and $\mu_{2}$, chemical potentials. The operation ( $\cdot$ ) denotes the scalar product and ( $\cdot$ ) convolution in both diad indices. The derivatives $d(a) / d t$ and $D(a) / D t$ have the form

$$
\begin{gather*}
\frac{d^{(a)}}{d t}(\ldots)=\frac{\partial}{\partial t}(\ldots)+\left(u_{a} \cdot \nabla\right)(\ldots) \\
\frac{D^{(a)}}{D t}(\ldots)=\frac{d^{(a)}}{d t}(\ldots)+[(\ldots) \cdot \nabla] u_{a}+(\ldots)\left(\nabla \cdot u_{a}\right) \tag{1.2}
\end{gather*}
$$

Given the normalized weights $\alpha_{k}$ and the thermodynamic functions, system (1.1) is adequate for finding the desired quantities. We write this system of equations below for specific values of $a_{k}$.

Mean Mass Flow Rate $\left(\alpha_{k}=c_{k}\right)$. In this case system (1.1) becomes

$$
\begin{gather*}
\frac{\partial \rho_{k}}{\partial t}+\nabla\left(\rho_{k} u_{m}\right)=-\nabla J_{k}^{m}, \quad \frac{\partial \rho}{\partial t}+\nabla \cdot\left(\rho u_{m}\right)=0, \\
\frac{\rho_{\mathrm{d}}^{(m)} \mathrm{u}_{\mathrm{m}}}{\mathrm{dt}}=\rho f-\nabla p+\nabla \cdot \tau_{m}, \quad \tau_{m}=\frac{\lambda_{m}}{T}\left(\nabla \cdot u_{m}\right) I+\frac{2 \mu_{m}}{T} e_{m}, \\
\frac{D^{(m) y_{1}^{m}}}{D t}=Q_{1}^{m}=-T \frac{\rho_{1} \rho_{2}}{\rho \alpha_{11}^{m}}\left\{J_{1}^{m}+\left[\alpha_{1}^{m}-\alpha_{11}^{m}\left(h_{1}-h_{2}\right)\right] \frac{\nabla T}{T^{2}}-\frac{1}{T} \alpha_{11}^{m}\left[\left(f_{1}-f_{2}\right)-\frac{\rho}{\rho_{2}}\left(\nabla \mu_{1}\right)_{p} \tau-\left(v_{1}-v_{2}\right) \nabla p\right]\right\},  \tag{1.3}\\
-q=\left[\alpha^{m}-\alpha_{1}^{m}\left(h_{1}-h_{2}\right)\right] \frac{\Delta T}{T^{2}}+\frac{1}{T} \alpha_{1}^{m}\left\{\left(f_{1}-f_{2}\right)-\frac{\rho}{\rho_{2}}\left(\nabla \mu_{1}\right)_{\rho, T}-\left(v_{1}-v_{2}\right) \nabla p-\frac{\rho}{\rho_{1} \rho_{2}} Q_{1}^{m}\right\}
\end{gather*}
$$

Mean Volume flow Rate $\left(\alpha_{k}=\rho_{k} v_{k}\right)$. If the mean volume flow rate is taken as the characteristic velocity $\mathrm{u}_{a}$, then we obtain in place of (1.3)

$$
\begin{gather*}
\frac{\partial \rho_{k}}{\partial t}+\nabla \cdot\left(\rho_{k} u_{v}\right)=-\nabla \cdot J_{k}^{v}, \quad \nabla \cdot u_{v}=\sum_{k=1}^{2} \rho_{k} \frac{d^{(k)} v_{k}}{d t}, \\
\rho \frac{d^{(v)} u_{v}}{d t}=\rho f-\nabla p+\nabla \cdot \tau_{v}-\sum_{k=1}^{2} \frac{D^{(v)} J_{k}^{v}}{D t}, \quad \tau_{v}=\frac{\lambda_{v}}{T}\left(\nabla \cdot u_{v}\right) I+\frac{2 \mu_{v}}{T} e_{v}  \tag{1.4}\\
-\frac{D^{(v)} J_{1}^{v}}{D t}=Q_{1}^{v}=-T \frac{\rho_{1} \rho_{2} v_{2}^{2}}{\alpha_{11}^{v}\left(\rho_{1} v_{1}^{2}+\rho_{2} v_{2}^{2}\right.} \\
\left.-\frac{1}{T} \alpha_{11}^{v}\left[\left(f_{1}^{v}-\frac{v_{1}}{v_{2}} f_{2}\right)-\left(1-\frac{v_{1}}{v_{2}}\right) \frac{d^{(v)} u_{v}}{d t}-\frac{1}{\rho_{2} v_{2}}\left(\nabla \mu_{1}^{v}-\alpha_{11}^{v}\left(h_{1}-\frac{v_{1}}{v_{2}} h_{2}\right)\right] \frac{\nabla^{2} T}{T^{2}}-\frac{v_{1}}{\rho_{2} v_{2}^{2}} \sum_{k=1}^{2} J_{k}^{v} \frac{d^{(v)} v_{k}}{d t}\right]\right\}, \\
-q=\left(1-\frac{v_{1}}{v_{2}}\right) J_{1}^{v} \varepsilon+\left[\alpha^{v}-\alpha_{1}^{v}\left(h_{1}-\frac{v_{1}}{v_{2}} h_{2}\right)\right] \frac{\nabla T}{T^{2}}+\frac{1}{T} \alpha_{1}^{v}\left(f_{1}-\frac{v_{1}}{v_{2}} f_{2}\right)- \\
\left.-\left(1-\frac{v_{1}}{v_{2}}\right) \frac{d^{(v)} u^{v}}{d t}-\frac{1}{\rho_{2} v_{2}}\left(\nabla \mu_{1}\right)_{p, r}-\frac{\rho_{1} v_{1}^{2}+\rho_{2} v_{2}^{2}}{\rho_{1} \rho_{2} v_{2}^{2}} Q_{1}^{v}-\frac{v_{1}}{\rho_{2} v_{2}^{2}} \sum_{k=1}^{2} J_{k}^{v} \frac{d^{(v)} v_{k}}{d t}\right]
\end{gather*}
$$

Let us note that the specific partial volumes $\mathrm{v}_{\mathrm{k}}$ can be considered constant [3] in many practical problems. Then system (1.4) simplifies. In particular, we obtain the following generalized compressibility condition:

$$
\begin{equation*}
\nabla u_{v} \equiv 0, \tag{1.5}
\end{equation*}
$$

which yields additional advantages in solving problems by using the mean volume flow rate.
Mean Molar Flow Rate ( $a_{\mathrm{k}}=\mathrm{N}_{\mathrm{k}} / \mathrm{N}$ ). The mean mixture molar flow rate $u_{\mu}$ is an important characteristic of mixture motion. By using it, the following system of equations can be obtained

$$
\begin{align*}
& \rho \frac{d^{(\mu)} u_{\mu}}{d t}=\rho f-\nabla p+\nabla \cdot \tau_{\mu}-\sum_{k=1}^{2} \frac{D^{(\mu)} J_{k}^{\mu}}{D t}, \\
& \tau_{i \mu}=\frac{\lambda_{\mu}}{T}\left(\nabla \cdot u_{\mu}\right) I+\frac{2 \mu_{\mu}}{T} \cdot e_{\mu}, \\
& \frac{D^{(\mu)} J_{1}^{\mu}}{D t}=Q_{1}^{\mu}=-T \frac{N_{1} N_{2} M_{1}^{2}}{\left(N_{1} M_{2}+N_{2} M_{1}\right) \alpha_{11}^{\mu}}\left\{J_{1}^{\mu}+\left[\alpha_{1}^{\mu}-\alpha_{11}^{\mu}\left(h_{1}-\frac{M_{2}}{M_{1}}{ }^{-} h_{2}\right) \frac{\nabla^{T}}{T}-\frac{1}{T} \alpha_{11}^{\mu}\left[\left(f_{1}-\frac{M_{2}}{M_{1}} f_{2}\right)-\right.\right.\right. \\
& \left.\left.-\left(1-\frac{M_{2}}{M_{1}}\right) \frac{d^{(\mu)} u_{\mu}}{d t}-\frac{N}{N_{2}}\left(\nabla \mu_{1}\right)_{r, T}-\left(v_{1}-\frac{M_{2}}{M_{1}} v_{2}\right) \nabla p\right]\right\},  \tag{1.6}\\
& -q=\left(1-M_{2_{2}}^{M_{1}}\right) J_{1}^{\mu} \mathrm{e}+\left[\alpha^{\mu}-\alpha_{1}^{\mu}\left(h_{1}-\frac{M_{2}}{M_{1}} h_{2}\right)\right] \frac{\nabla^{T}}{T^{2}}+ \\
& +\frac{1}{T} \alpha_{1}^{\mu}\left[\left(f_{1}--\frac{M_{2}}{M_{1}}-f_{2}\right)-\left(1-\frac{M_{2}}{M_{1}}\right) \frac{d^{(\mu)} u_{\mu}}{d t}-\frac{N}{N_{2}}\left(\nabla \mu_{1}\right)_{0} T-\left(v_{1}-\frac{M_{2}}{M_{1}} v_{2}\right) \nabla^{p}-\frac{N_{2} M_{1}+N_{1} M_{2}}{N_{1} N_{2} M_{1}^{2}} Q_{1}^{\mu}\right] .
\end{align*}
$$

Velocity of the Second Component $\left(\alpha_{k}=\delta_{k 2}\right)$. Cases exist when it is most convenient to use the velocity of one of the components as the characteristic mixture velocity. Let this velocity be $u_{2}$. Then we have the following systen to find the desired quantities

$$
\begin{gather*}
\partial{\rho_{1}}_{\partial t}+\nabla \cdot\left(\rho_{1} u_{2}\right)=-\nabla^{J_{1}}, \quad \frac{\partial \rho_{2}}{\partial t}+\nabla \cdot\left(\rho_{2} u_{2}\right)=0, \\
\frac{\rho d^{(2)} u_{2}}{d t}=\rho f-\nabla \rho+\nabla \cdot \tau_{2}-\frac{D^{(2)} J_{1}}{D t}, \quad \tau_{2}=\frac{\lambda}{T}\left(\nabla \cdot u_{2}\right) I+\frac{2 \mu}{T} e_{2}, \\
\frac{D^{(2)} J_{1}}{D t}=Q_{1}=-T \frac{\rho_{1}}{\alpha_{11}}\left\{J_{1}+\left(\alpha_{1}-\alpha_{11} h_{1}\right) \frac{\nabla T}{T^{2}}-\frac{1}{T} \alpha_{11}\left[f_{1}-\nabla\left(\mu_{1}\right)_{\rho, T}-v_{1} \nabla \rho-\frac{d^{(2)} u_{2}}{d t}\right]\right\},  \tag{1.7}\\
-q=J_{1} \varepsilon+\left(\alpha-\alpha_{1} h_{1}\right)-\frac{\nabla T}{T^{2}}+\frac{1}{T} \alpha_{1}\left[f_{1}-\left(\nabla \mu_{1}\right)_{p, T}-v_{1} \nabla p-\frac{d^{(2)} u_{2}}{d t}-\frac{Q_{1}}{\rho_{1}}\right] .
\end{gather*}
$$

## 2. Motion of a Binary Fluid Mixture within an Infinite Cylinder

To illustrate the theory developed, let us examine the motion of a binary fluid mixture within an infinite rotating cylinder at a constant temperature and in the absence of external forces, as the simplest example. Moreover, the specific partial volumes will be considered constant.

Let an infinite cylinder rotate around its $z$ axis at the constant angular velocity $\Omega$, and let $\rho_{1}^{o}$ and $\rho_{2}^{\circ}$ be the densities of the components at the initial time. Find the distribution of these densities at later times.

It is most convenient to use the mean volume flow rate $u_{v}$ to solve this problem since it is directed only in the $\theta$ direction in an ( $r, \theta, z$ ) cylindrical coordinate system. At the same time, other characteristic velocities are directed along both the $\theta$ and the $r$ axes, which radically complicates the solution of the formulated problem.

The complete system of equations describing such mixture motion can be obtained from (1.4)-(1.5)

$$
\begin{gather*}
u_{v}=\left\{0 ; u_{\theta}(r, t) ; 0\right\}, \quad J_{1}^{v}=J=\left\{J_{r}(r, t) ; J_{\theta}(r, t) ; 0\right\}, \\
\frac{\partial \rho_{1}}{\partial t}=-\frac{1}{r} \frac{\partial}{\partial r}\left(r J_{r}\right), \rho_{2}=\frac{1}{v_{2}}\left(1-\rho_{1} v_{1}\right), \\
\frac{\partial u_{\theta}}{\partial t}=v \frac{\partial}{\partial r}\left[\frac{1}{r} \frac{\partial}{\partial r}\left(r u_{\theta}\right)\right]-\frac{1}{\rho}\left(1-\frac{v_{1}}{v_{2}}\right)\left[\frac{\partial J_{\theta}}{\partial t}+J_{r}\left(\frac{u_{\theta}}{r}+\frac{\partial u_{\theta}}{\partial r}\right)\right], \\
\frac{\partial J_{r}}{\partial t}-2 \frac{J_{\theta} u_{\theta}}{r}=-T \frac{\rho_{1} \rho_{2} v_{2}^{2}}{\left(\rho_{1} v_{1}^{2}+\rho_{2} v_{2}^{2}\right) \alpha_{1}^{v}}-\left\{J_{r}+\frac{\alpha_{11}^{v}}{T}\left[-\left(1-\frac{v_{1}}{v_{2}}\right) \frac{u_{\theta}^{2}}{r}+\frac{\mu_{41}}{\rho_{2} v_{2}} \frac{\partial \rho_{1}}{\partial r}\right]\right\},  \tag{2.1}\\
\frac{\partial J_{\theta}}{\partial t}+J_{r}\left(\frac{u_{\theta}}{r}+-\frac{\partial u_{\theta}}{\partial r}\right)=-T \frac{\rho_{1} \rho_{2} v_{2}^{2}}{\rho_{0}}\left[\begin{array}{c}
\left.\rho_{1}^{2}+\rho_{2} v_{2}^{2}\right) \alpha_{11}^{v}
\end{array} J_{\theta}+\frac{\alpha_{11}^{p}}{T}\left(1-\frac{v_{1}}{v_{2}}\right) \frac{\partial u_{\theta}}{\partial t}\right\}, \\
\frac{1}{\rho} \frac{\partial \rho}{\partial r}=-\frac{u_{\theta}^{2}}{r}-\frac{1}{\rho}\left(1-\frac{v_{1}}{v_{2}}\right)\left(\frac{\partial J_{r}}{\partial t}-2 \frac{J_{\theta} u_{\theta}}{r}\right) .
\end{gather*}
$$

The system of nonlinear partial differential equations (2.1) with the known phenomenological constants is sufficient for finding the six unknowns $\rho_{1}, \rho_{2}, u_{\theta}, J_{\theta}, J_{r}$, and $p$.

As $t \rightarrow \infty$, the following known results [3] can be obtained from (2.1):

$$
\begin{gather*}
u_{\theta}=\Omega r ; \quad J_{\theta}=J_{r}=0, \quad \frac{d p}{d r}=\rho \Omega^{2} r,  \tag{2.2}\\
\frac{d \rho_{1}}{d r}=\frac{1}{\mu_{11}} \rho_{2}\left(v_{2}-v_{1}\right) \Omega^{2} r, \quad \rho_{2}=\frac{1}{v_{2}}\left(1-\rho_{1} v_{1}\right) .
\end{gather*}
$$

In the nonstationary case, we linearize (2.1) to obtain the analytic solution and we assume that the diffusion flux does not affect the mixture velocity distribution $u_{\theta}$. We consequently obtain

$$
\begin{gather*}
\frac{\partial u_{\theta}}{\partial t}=v \frac{\partial}{\partial r}\left[r \frac{\partial}{\partial r}\left(r u_{\theta}\right)\right], \quad \frac{\partial \rho_{1_{2}}}{\partial t}=-\frac{1}{r} \frac{\partial}{\partial r}\left(r J_{r}\right), \\
\frac{\partial J_{r}}{\partial t}+K J_{r}+K D \frac{\partial \rho_{i}}{\partial r}=K L \frac{u_{\theta}^{2}}{r}, \quad \rho_{2}=\frac{1}{v_{2}}\left(1-\rho_{1} v_{1}\right),  \tag{2.3}\\
\frac{\partial J_{\theta}}{\partial t}+K J_{\theta}=-K L \frac{\partial u_{\theta}}{\partial t}, \quad \frac{\partial \rho}{\partial r}=\rho \frac{u_{\theta}^{2}}{r}-\left(1-\frac{v_{1}}{v_{2}}\right)-\frac{\partial J_{r}}{\partial t^{-}} .
\end{gather*}
$$

The constants $\mathrm{K}, \mathrm{D}, \mathrm{L}$ in (2.3) have the form

$$
\begin{equation*}
K=T \frac{\rho_{1} \rho_{2} v_{2}^{2}}{\left(\rho_{1} v_{1}^{2}+\rho_{2} v_{2}^{2}\right) \alpha_{11}^{v}}, \quad D=\frac{\alpha_{11}^{v} \mu_{11}^{\rho}}{T \rho_{2} v_{2}}, \quad L=\frac{1}{T} \alpha_{11}^{v}\left(1-\frac{v_{1}}{v_{2}}\right) . \tag{2.4}
\end{equation*}
$$

Let us note that we retain the term $u_{\theta} / r$ in (2.3) since it causes a redistribution of components along the $r$ axis.

With conditions $u_{\theta}=0$ at $t=0$ and $u_{\theta}=\Omega R$ at $r=R$, the first equation of (2.3) yields the solution [4]

$$
\begin{equation*}
u_{0}(r, t)=\Omega r+2 \Omega R \sum_{n=1}^{\infty} \frac{I_{1}\left(\lambda_{n} \frac{r}{R}\right)}{\lambda_{n} I_{0}\left(\lambda_{n}\right)} \exp \left(-\lambda_{n}^{2} \frac{v t}{R^{2}}\right) \tag{2.5}
\end{equation*}
$$

Here $I_{0}$ and $I_{1}$ are Bessel functions of the first kind of zero and first order, respectively, and $\lambda_{n}$ are positive values satisfying $I_{1}(\lambda)=0$.

We henceforth limit ourselves to the following approximation for $u_{\theta}^{2}$ :

$$
u_{0}^{2} \cong \Omega^{2} r^{2}+2 \Omega^{2} R r \sum_{n=1}^{\infty} \frac{I_{1}\left(\lambda_{n} \frac{r}{R}\right)}{\lambda_{n} I_{0}\left(\lambda_{n}\right)} \exp \left(-\lambda_{n}^{2} \frac{v t}{R^{2}}\right)
$$

Then the equation to find $\rho_{1}, J_{0}, J_{r}$ has the form

$$
\begin{equation*}
\frac{\partial \rho_{1}}{\partial t}=-\frac{1}{r} \frac{\partial}{\partial r}\left(r J_{r}\right), \quad \frac{\partial J_{\theta}}{\partial t}+K J_{\theta}=2 K L \Omega v \sum_{n=1}^{\infty} \frac{\lambda_{n} I_{1}\left(\lambda_{n} \frac{r}{R}\right)}{R I_{0}\left(\lambda_{n}\right)} \exp \left(-\lambda_{n}^{2} \frac{v t}{R^{2}}\right) \tag{2.6}
\end{equation*}
$$

$$
\frac{\partial J_{r}}{\partial t}+K J_{r}+K D \frac{\partial \rho_{1}}{\partial r}=K L \Omega^{2}\left[r+2 R \sum_{n=1}^{\infty} \frac{I_{1}\left(\lambda_{n} \frac{r}{R}\right)}{\lambda_{n} I_{0}\left(\lambda_{n}\right)} \exp \left(-\lambda_{n}^{2} \frac{v t}{R^{2}}\right) .\right.
$$

If $K \rightarrow \infty$ in (2.6) (this case corresponds to neglecting $\partial J / \partial t$ in classical diffusion theory), then system (2.6) together with the conditions $J_{r}=0$ at $t=0$ and $J_{r}=0$ at $r=R$ yields a solution of the form

$$
\begin{gather*}
J_{\theta}=2 L \Omega v \sum_{n=1}^{\infty} \frac{\lambda_{n}}{R} \frac{I_{1}\left(\lambda_{n} \frac{r}{R}\right)}{I_{0}\left(\lambda_{n}\right)} \exp \left(-\lambda_{n}^{2} \frac{v t}{R^{2}}\right), \\
J_{r}=\frac{2 L \Omega^{2} v R}{v-D} \sum_{n=1}^{\infty} \frac{I_{i}\left(\lambda_{n} \frac{r}{R}\right)}{I_{0}\left(\lambda_{n}\right)}\left\{\exp \left(-\lambda_{n}^{2} \frac{v t}{R^{2}}\right)-\exp \left(-\lambda_{n}^{2} \frac{D t}{R^{2}}\right)\right\} \\
+\frac{\rho_{i}=\rho_{1}^{0}+\frac{L \Omega^{2}}{2 D}\left(r^{2}-\frac{R^{2}}{2}\right)+}{v-D} \sum_{n=1}^{\infty} \frac{I_{0}\left(\lambda_{n} \frac{r}{R}\right)}{\lambda_{n}^{2} I_{0}\left(\lambda_{n}\right)}\left\{\frac{1}{v} \exp \left(-\lambda_{n}^{2} \frac{v t}{R^{2}}\right)-\frac{1}{D} \exp \left(-\lambda_{n}^{2} \frac{D t}{R^{2}}\right)\right\}
\end{gather*}
$$

Under the condition $D \ll v$, we have for large $t$

$$
\begin{equation*}
\rho_{1}=\rho_{1}^{0}+\frac{L \Omega^{2}}{2 D}\left(r^{2}-\frac{R^{2}}{2}\right)-\frac{2 L \Omega R^{2}}{D} \sum_{n=1}^{\infty} \frac{I_{0}\left(\lambda_{n}^{r} r^{-}\right)}{\lambda_{n}^{2} I_{0}\left(\lambda_{n}\right)} \exp \left(-\lambda_{n}^{2} \frac{D t}{R^{2}}\right) \tag{2.8}
\end{equation*}
$$

If $K \neq \infty$ in (2.6), i.e., it is impossible to neglect the term $\partial J_{\theta} / \partial t$, then system (2.6) must be solved in combination with the conditions $J_{r}=\partial J_{r} / \partial t=J_{\theta}=0$ for $t=0$, $J_{r}=0$ for $r=R$.

In this case, for $K^{2}-4 K D\left(\lambda_{n}^{2} / R^{2}\right)=\mu_{n}^{2} \geq 0$ we find the following expression for $\rho_{1}$ :

$$
\begin{gather*}
\rho_{1}=\rho_{1}^{2}+\frac{L}{2 D} \Omega^{2} R^{2}\left(-\frac{r^{2}}{R^{2}}-\frac{1}{2}\right)+\frac{2 K L \Omega^{2} v}{R^{2}} \sum_{n=1}^{\infty} \frac{\lambda_{n}^{2} I_{0}\left(\lambda_{n} \frac{r}{R}\right)}{\mu_{n} I_{0}\left(\lambda_{n}\right)}\left(\frac{1}{-\lambda_{n}^{2}} \frac{1}{R^{2}} v+\frac{1}{2}\left(k-\mu_{n}\right)\right. \\
\times\left[-\frac{R^{2}}{\lambda_{n}^{2} v} \exp \left(-\lambda_{n}^{2} \frac{v t}{R^{2}}\right)+\frac{2}{K-\mu_{n}} \exp \left(-\frac{K-\mu_{n}}{2} t\right)\right]- \\
-  \tag{2.9}\\
-\frac{1}{-\frac{\lambda_{n}^{2}}{R^{2}} v+\frac{1}{2}\left(K+\mu_{n}\right)}\left[-\frac{R^{2}}{\lambda_{n}^{2} v} \exp \left(-\lambda_{n}^{2} \frac{v t}{R^{2}}\right)+\frac{2}{K+\mu_{n}} \exp \left(-\frac{K+\mu_{n}}{2} t\right)\right] .
\end{gather*}
$$

The solution (2.9) differs from (2.7) only at the initial times, while (2.9) and (2.7) tend to a common distribution as $t \rightarrow \infty$, of the form

$$
\rho_{1}=\rho_{1}^{0}+\frac{\rho_{2}^{0}\left(v_{2}-v_{1}\right)}{\mu_{11}^{\rho}} \Omega^{2} R^{2}\left(\frac{r^{2}}{R^{2}}-\frac{1}{2}\right)
$$

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